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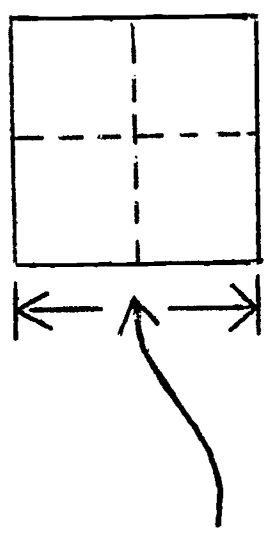
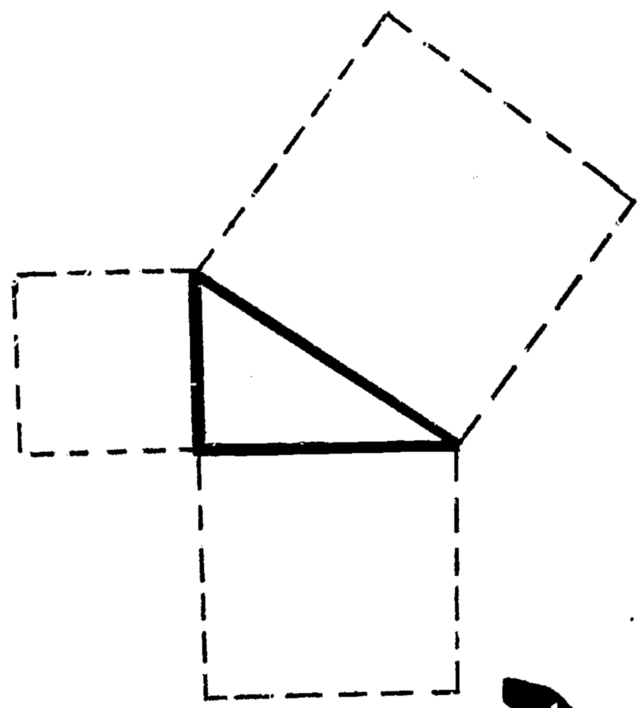
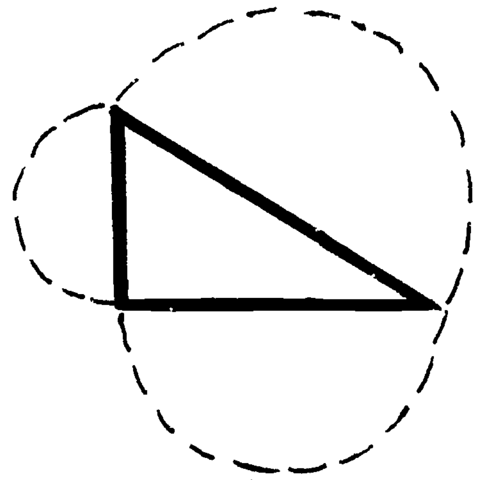
This booklet, one of a series, has been developed for the project, A Program for Mathematically Underdeveloped Pupils. A project team, including inservice teachers, is being used to write and develop the materials for this program. The materials developed in this booklet include (1) elementary properties of the exponent, (2) properties of the right triangle, (3) squares and square roots, (4) the Pythagorean Theorem, (5) the acute and the obtuse triangle, (6) length of a diagonal of a rectangular solid, and (7) activities involving the determination of areas formed by the squares of sides of triangles. Accompanying these booklets will be a "Teaching Strategy Booklet" which will include a description of teacher techniques, methods, suggested sequences, academic games, and suggested visual materials. (RP)

# SQUARES





# SQUARE ROOTS



# RIGHT TRIANGLES

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# SQUARES, SQUARE ROOTS, RIGHT TRIANGLES

Table of Contents

# SQUARES, SQUARE ROOTS, AND RIGHT TRIANGLES

## Introduction - Exponents

The product 3 X 3 may be written as $3^2$. The number 2 is called an exponent, and 3 is called the base. Other examples are:

$$4 \times 4 = 4^2$$

$$7 \times 7 = 7^2$$

Some examples of exponents other than 2 are:

$$4 \times 4 \times 4 = 4^3$$

$$3 \times 3 \times 3 \times 3 = 3^4$$

$$5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^6$$

Of course, $4^3$ equals 64, $3^4$ equals 81, and $5^6$ equals 15,625.

### Activities

In each of the problems 1 - 5 write the correct exponent in the "box":

1. $9 \times 9 = 9^{\square}$
2. $14 \times 14 = 14^{\square}$
3. $2 \times 2 \times 2 \times 2 \times 2 = 2^{\square}$
4. $6 \times 6 \times 6 \times 6 \times 6 \times 6 = 6^{\square}$
5. $7^{\square} = 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7$

In each of the problems 6 - 10, give the correct base numbers:

6. $8 \times 8 \times 8 = \square^3$
7. $13 \times 13 = \square^2$
8. $9 \times 9 \times 9 \times 9 = \square^4$
9. $126 \times 126 \times 126 = \square^3$
10. $1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 = \square^7$

Compute the value for each of the following:

Example: $3^3 = (3 \times 3) \times 3$

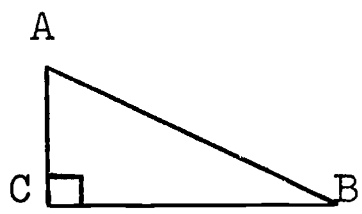
$= 9 \times 3$

$= 27$

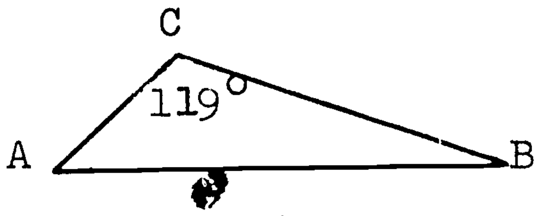
| | | |
|---|---|---|
| 11. $7^2$ | 16. $1^5$ | 21. $5^3$ |
| 12. $8^2$ | 17. $1^6$ | 22. $10^2$ |
| 13. $2^3$ | 18. $1^{10}$ | 23. $10^3$ |
| 14. $2^5$ | 19. $0^2$ | 24. $10^4$ |
| 15. $4^3$ | 20. $0^3$ | 25. $8^4$ |

## The Right Triangle

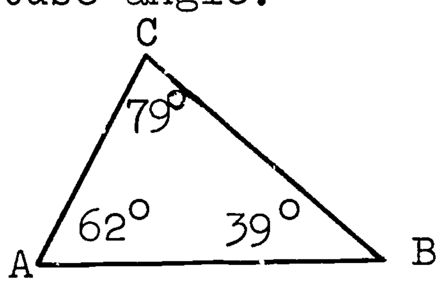
A right triangle is a triangle with one right angle. The side opposite the right angle is the hypotenuse. Angle C of the right triangle illustrated below is the right angle and $\overline{AB}$ is the hypotenuse. An acute triangle is one with each of its 3 angles acute, and an obtuse triangle has exactly one obtuse angle.



Right triangle



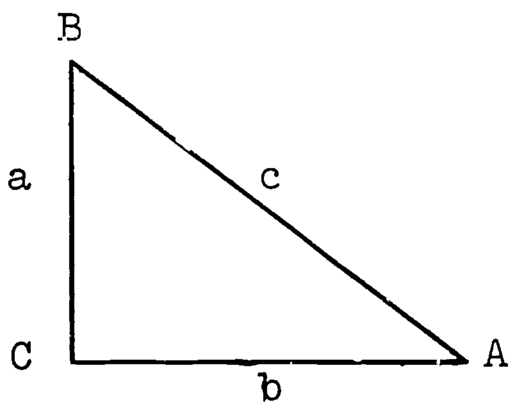
Obtuse triangle



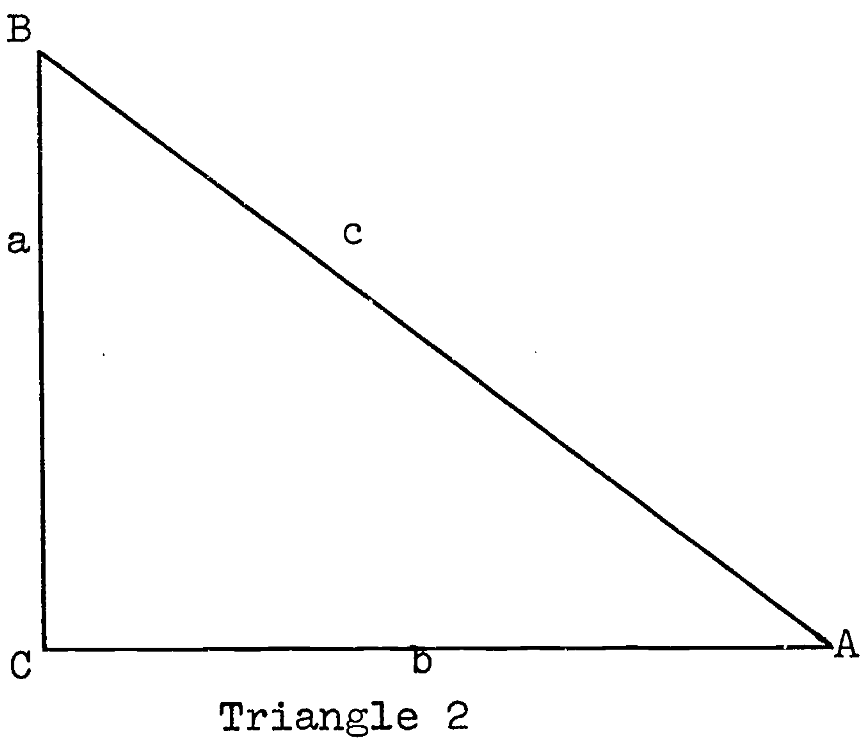
Acute triangle

## Activities

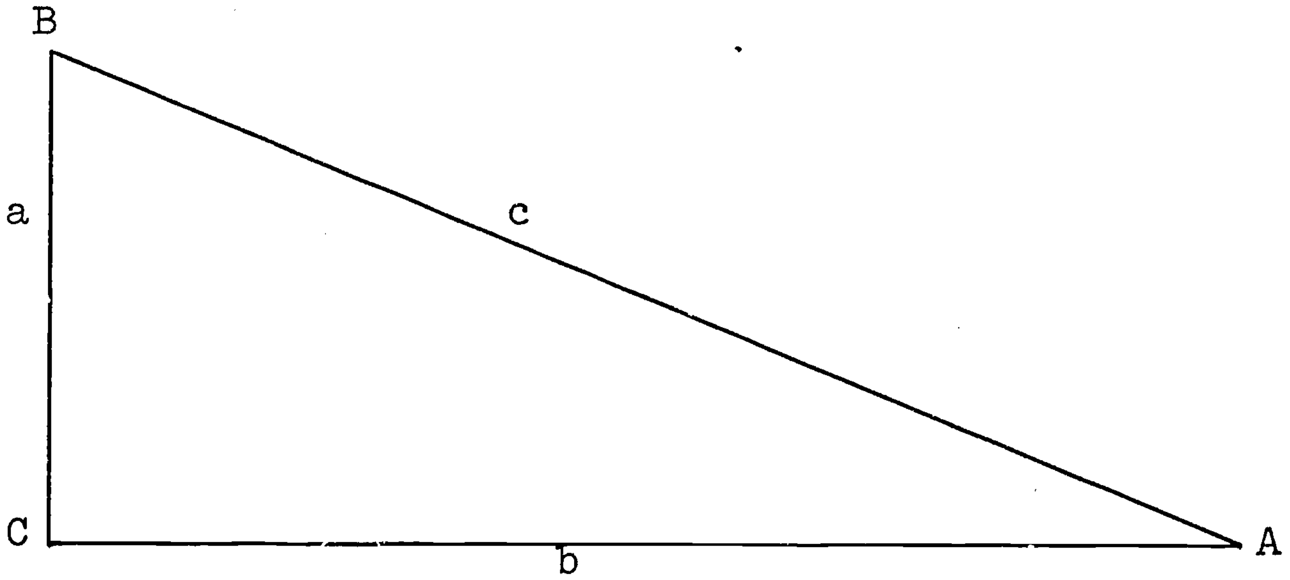
1. Use your ruler to find the lengths of the three sides in each of the following triangles and then complete table 1. Measure to the nearest centimeter.



Triangle 1



Triangle 2




Triangle 3

| Triangle | a | b | c | $a^2$ | $b^2$ | $c^2$ |
|---|---|---|---|---|---|---|
| 1 | | | | | | |
| 2 | | | | | | |
| 3 | | | | | | |

( Table 1 )

2. Observe the $a^2$, $b^2$, and $c^2$ columns for triangle 1. Do you see a relationship among $a^2$, $b^2$, and $c^2$? ______________

3. Is $\angle C$ of triangle 1 acute, right or obtuse? (You may use your protractor.) ____________

4. Is triangle 1 acute, right, or obtuse? ______________

5. Observe the $a^2$, $b^2$, and $c^2$ columns for triangle 2. Do you see a relationship among $a^2$, $b^2$, and $c^2$ for triangle 2? __________

6. Is $\angle C$ of triangle 2 acute, right, or obtuse? ____________

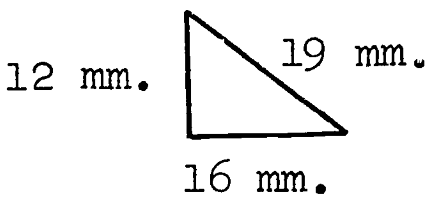
7. Is triangle 2 acute, right, or obtuse? ______________

8. Observe the $a^2$, $b^2$, and $c^2$ columns for triangle 3. Do you see a relationship among $a^2$, $b^2$, and $c^2$ for triangle 3? __________
Try to write a mathematical statement using $a^2$, $b^2$, and $c^2$ which expresses this relationship. ________________________

9. Is triangle 3 acute, right or obtuse? ________

In each of the problems 10 - 17, the lengths of the three sides of a triangle are given. Indicate which are right triangles.

10.

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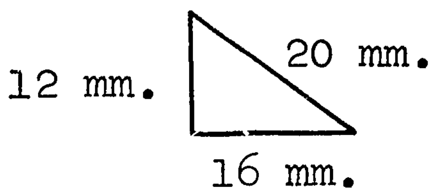
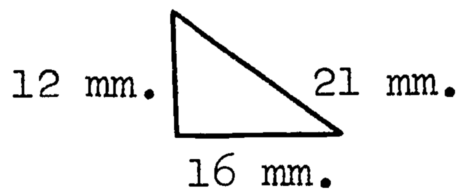



12.




13. 9, 12, and 15 inches ________

14. 10, 24, and 26 centimeters ________

15. 16, 20, and 28 millimeters ________

16. 21, 28, and 35 inches ________

17. 24, 35, and 40 inches ________

<u>Squares and Square Roots</u>

Multiplying 4 by 4 and obtaining the result, 16, is called "squaring 4." In other words $4^2$ is read "4 squared." Suppose you wanted to reverse the process. This would be finding a number that when multiplied by itself would give 16. That is, the square root of 16 is 4. In symbols, $\sqrt{16} = 4$ where " $\sqrt{\ }$ " means "the square root of."

Compare the two:

Find the square of 4: $4^2 = 4 \times 4 = 16$

Find the square root of 16: $\sqrt{16} = \sqrt{4 \times 4} = 4$

The meaning of square root may also be expressed by introducing the word "factor." The factors of a given number are the numbers listed in an indicated product.

Two factors of 10 are 5 and 2 because 5 X 2 = 10. Two factors of 48 are 12 and 4. The number 48 may be factored into pairs of numbers other than 12 and 4.

These other factor pairs are: 16 and 3
8 and 6
2 and 24
48 and 1

Do you see that the product of each pair is 48? The number 48 has factors other than the factor pairs listed above. What about 12 X 2 X 2 or 4 X 4 X 3? However, we are interested in special factor pairs at this time. These special factor pairs are equal factor pairs.

Activities

1. Examine the following table and then complete the sentence which defines the square root of a number.

| Number | Number Factored Into Equal Factors | Square Root of the Number |
|---|---|---|
| $6^2$ | 6 X 6 | 6 |
| $3^2$ | 3 X 3 | 3 |
| $25^2$ | 25 X 25 | 25 |
| $16^2$ | 16 X 16 | 16 |
| 4 | 2 X 2 | 2 |
| 144 | 12 X 12 | 12 |
| 169 | 13 X 13 | 13 |

The square root of a certain number is one of the two

__________ __________ of that certain number.

Use the following Table of Squares to give the square root of the numbers in problems 2 - 10.

Table of Squares

| | | | | | |
|---|---|---|---|---|---|
| $1^2 = 1 \times 1 = 1$ | $16^2 = 16 \times 16 = 256$ |
| $2^2 = 2 \times 2 = 4$ | $17^2 = 17 \times 17 = 289$ |
| $3^2 = 3 \times 3 = 9$ | $18^2 = 18 \times 18 = 324$ |
| $4^2 = 4 \times 4 = 16$ | $19^2 = 19 \times 19 = 361$ |
| $5^2 = 5 \times 5 = 25$ | $20^2 = 20 \times 20 = 400$ |
| $6^2 = 6 \times 6 = 36$ | $21^2 = 21 \times 21 = 441$ |
| $7^2 = 7 \times 7 = 49$ | $22^2 = 22 \times 22 = 484$ |
| $8^2 = 8 \times 8 = 64$ | $23^2 = 23 \times 23 = 529$ |
| $9^2 = 9 \times 9 = 81$ | $24^2 = 24 \times 24 = 576$ |
| $10^2 = 10 \times 10 = 100$ | $25^2 = 25 \times 25 = 625$ |
| $11^2 = 11 \times 11 = 121$ | $26^2 = 26 \times 26 = 676$ |
| $12^2 = 12 \times 12 = 144$ | $27^2 = 27 \times 27 = 729$ |
| $13^2 = 13 \times 13 = 169$ | $28^2 = 28 \times 28 = 784$ |
| $14^2 = 14 \times 14 = 196$ | $29^2 = 29 \times 29 = 841$ |
| $15^2 = 15 \times 15 = 225$ | $30^2 = 30 \times 30 = 900$ |

2. 121
3. 529
4. 484
5. 576
6. 169
7. 324
8. 81
9. 8100
10. 4900

11. From the Table of Squares, can you find any "squares" that end in 2, 3, 7 or 8? __________

12. Divide some of the "squares" by 3 and see if the remainder is always a 0 or a 1. __________

    Do you think you could ever get a 2 remainder? __________

13. 67 X 67 = 4489. What is $\sqrt{4489}$ ? __________

14. $2.5 \times 2.5 = 6.25$. Find $\sqrt{6.25}$ ________

15. $57 \times 57 = 3249$. $\sqrt{3249}$ = ________

16. $36^2 = 1296$. $\sqrt{1296}$ = ________

17. $8.75^2 = 76.5625$. Find $\sqrt{76.5625}$. ________

18. $\sqrt{163.84} = 12.8$. What is $12.8 \times 12.8$? ________

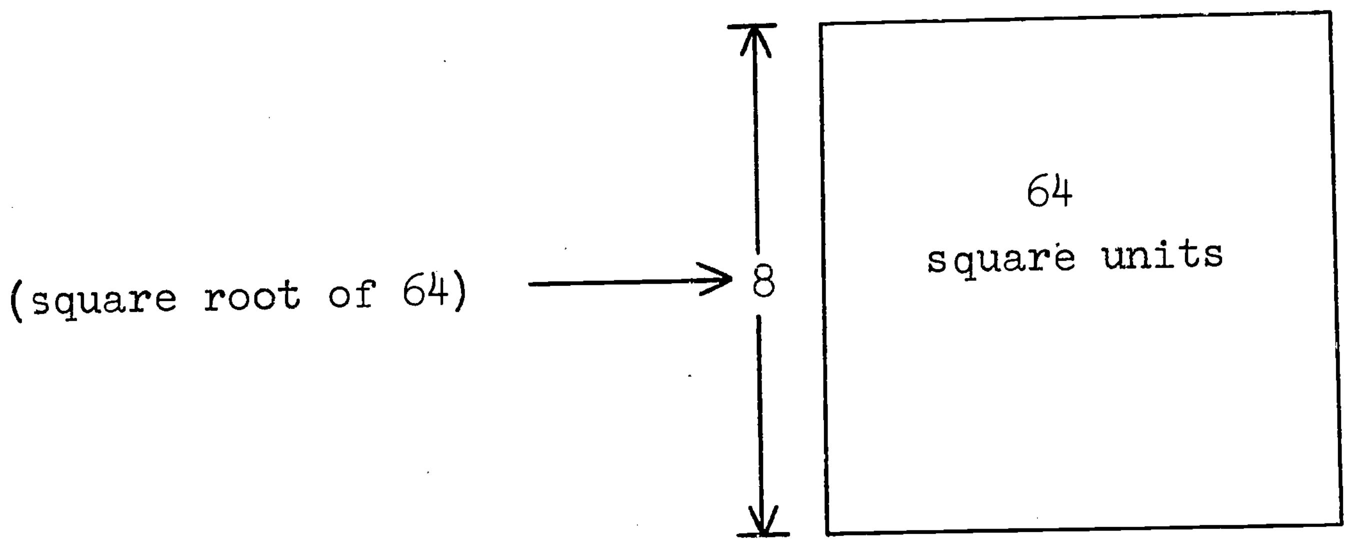
19. $\sqrt{2401} = 49$. What is $49 \times 49$? ________

20. $\sqrt{3364} = 58$. What is $58^2$ ? ________

21. $\sqrt{53.29} = 7.3$. $7.3^2$ = ________

22. $\sqrt{196} = 14$. $14^2$ = ________

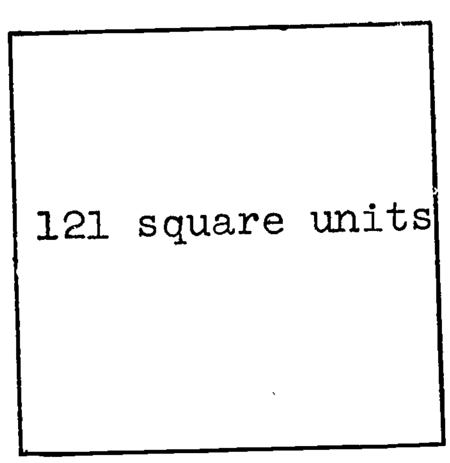
The "square root" of a number can be illustrated in terms of geometry. Consider a "square" that has a measure of 64 square units, then the length of one side of the square is the square root of 64.



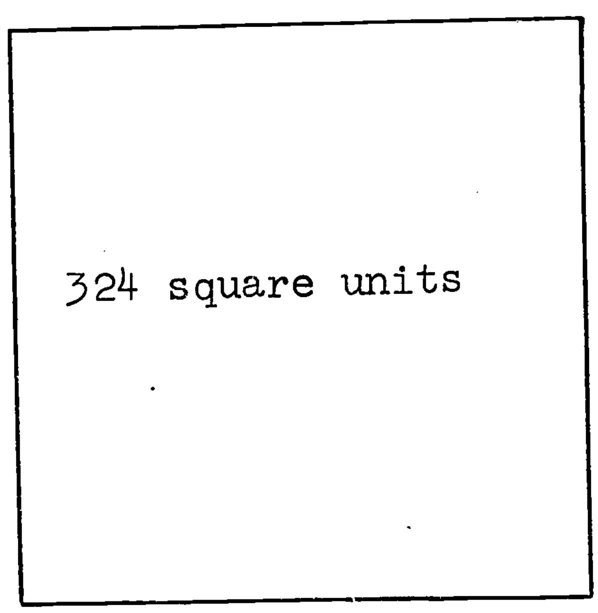

Activities

Use your Table of Squares to find the length for one side (square root) of each square.

1.




2.



3. $\boxed{\text{1 square unit}}$

<u>Example</u>: Solve the following open sentence.

$$c^2 = 3^2 + 4^2$$

<u>Solution</u>:

$$c^2 = 9 + 16$$
$$c^2 = 25$$
$$c \times c = 5 \times 5$$
$$c = 5$$

## Activities

Solve each of the problems 1 - 12 as in the preceding example. Use the Table of Squares on page 6 if you need it.

1. $c^2 = 5^2 + 12^2$
   $c^2 =$ ________
   $c =$ ________

2. $c^2 = 8^2 + 15^2$
   $c^2 =$ ________
   $c =$ ________

3. $c^2 = 9^2 + 12^2$
   $c^2 =$ ________
   $c =$ ________

4. $c^2 = 7^2 + 24^2$
   $c^2 =$ ________
   $c =$ ________

5. $c^2 = 12^2 + 16^2$
   $c^2 =$ ________
   $c =$ ________

6. $c^2 = .6^2 + .8^2$
   $c^2 =$ ________
   $c =$ ________

7. $c^2 = 1^2 + 2.4^2$
   $c^2 =$ ________
   $c =$ ________

8. $c^2 = .15^2 + .2^2$
   $c^2 =$ ________
   $c =$ ________

9. $c^2 = (\frac{3}{5})^2 + (\frac{4}{5})^2$

   $c^2 =$ ____________

   $c =$ ____________

10. $c^2 = 1^2 + (\frac{12}{5})^2$

   $c^2 =$ ____________

   $c =$ ____________

11. $c^2 = (\frac{1}{4})^2 + (\frac{3}{5})^2$

   $c^2 =$ ____________

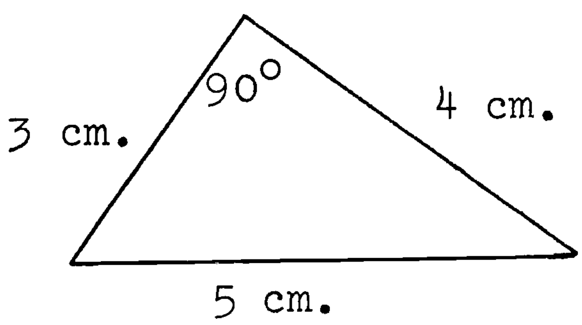
   $c =$ ____________

12. $c^2 = 5^2 + 6^2$

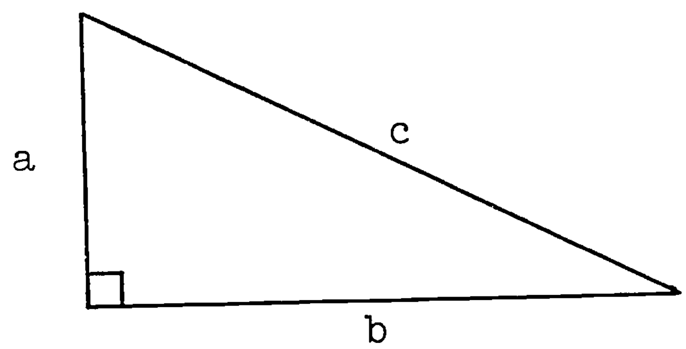
   $c^2 =$ ____________

   $c =$ ____________

In each of the right triangles 1 - 3 represented on pages 2 and 3, $c^2 = a^2 + b^2$ where a, b, and c are the lengths of the sides of each of the triangles. This is probably the most famous of all mathematical statements. It is the <u>Pythagorean Theorem</u>. This theorem asserts, in a right triangle the square of the length of the hypotenuse equals the sum of the squares of the lengths of the other two sides. This is illustrated.




$5^2 = 3^2 + 4^2$



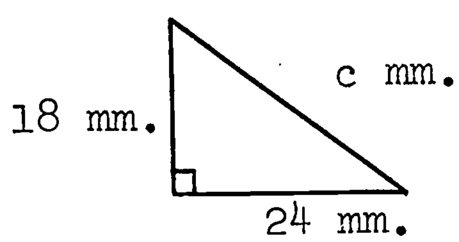

$c^2 = a^2 + b^2$

<u>Activities</u>

Solve the following problems by using the Pythagorean Theorem.

1. In the right triangle pictured here find the length of the hypotenuse.

   $c =$ ____________

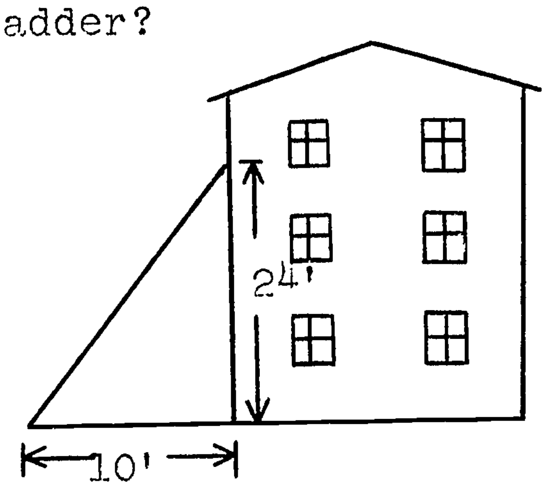



2. Find the length of the hypotenuse of a right triangle given that the lengths of the other two sides are 8 mm. and 15 mm.

   $c =$ ____________

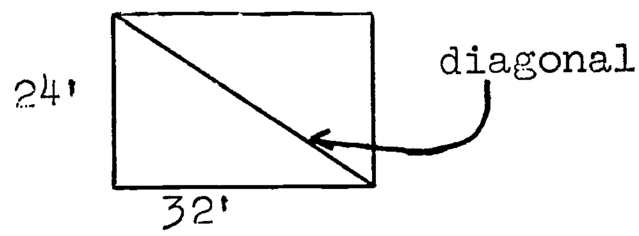
3. Find the length of the hypotenuse of a right triangle given that the lengths of the other two sides are 7 cm. and 24 cm.

4. A ladder is leaning against a building. The top of the ladder touches the building 24 feet above the ground. The foot of the ladder is 10 feet from the building. What is the length of the ladder?

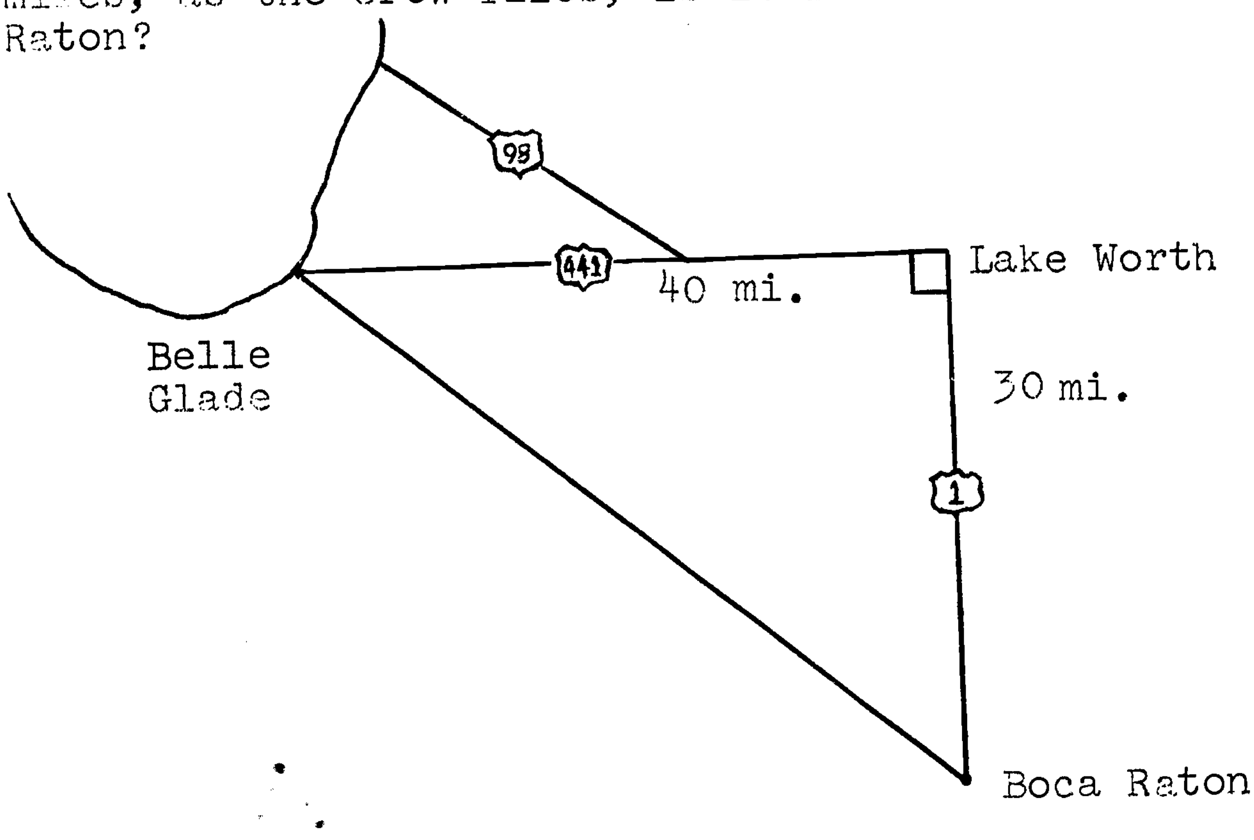



5. Suppose you walked 9 yards north and then 12 yards east. How far would you be from your starting point?

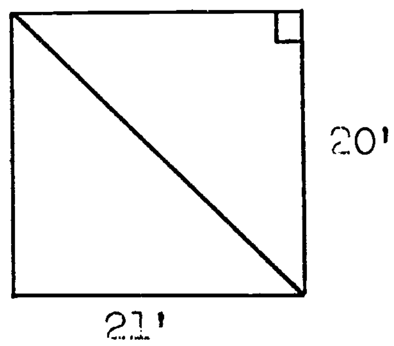
6. How much shorter is it to travel diagonally from corner to corner across a vacant lot which measures 24 feet by 32 feet than it is to go around the edge?



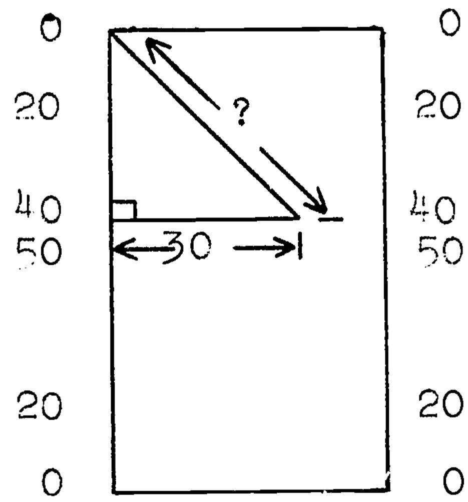

7. The distance from Lake Worth to Belle Glade is 40 miles, and the distance from Lake Worth to Boca Raton is 30 miles. How many miles, as the crow flies, is it from Belle Glade to Boca Raton?



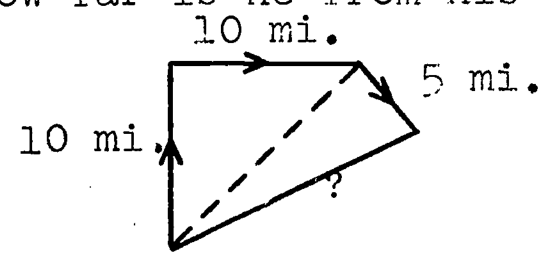
8. A room is 21 feet long and 20 feet wide. How far is it from one corner to the opposite corner?




9. John caught a pass on the 40 yard line of the opposing team 30 yards from the left sideline. He was forced to run in a straight line to the extreme left end of the goal to make a touchdown. How far did he run?




10. A man travels 10 miles north, 10 miles east, and then 5 miles southeast. How far is he from his starting point?




11. Traveling from this point on earth a man always goes south. Where is the man?

Example: Solve the following open sentence.

$$10^2 = a^2 + 8^2$$

Solution: $100 = a^2 + 64$ (What number when added to 64 yields a sum of 100?)

$$a^2 = 100 - 64$$

$$a^2 = 36$$

$$a \times a = 6 \times 6$$

$$a = 6$$

## Activities

Solve each of the problems 1 - 18 as in the preceding example. Use the Table of Squares on page 6 if you need it.

1. $10^2 = 6^2 + b^2$

   $b^2 =$ __________

   $b =$ __________

2. $13^2 = a^2 + 12^2$

   $a^2 =$ __________

   $a =$ __________

3. $34^2 = a^2 + 30^2$

   $a^2 =$ __________

   $a =$ __________

4. $39^2 = 36^2 + b^2$

   $b^2 =$ __________

   $b =$ __________

5. $45^2 = a^2 + 36^2$

   $a^2 =$ __________

   $a =$ __________

6. $.5^2 = .3^2 + b^2$

   $b^2 =$ __________

   $b =$ __________

7. $1.3^2 = .5^2 + b^2$

   $b^2 =$ __________

   $b =$ __________

8. $(\frac{5}{6})^2 = (\frac{1}{2})^2 + b^2$

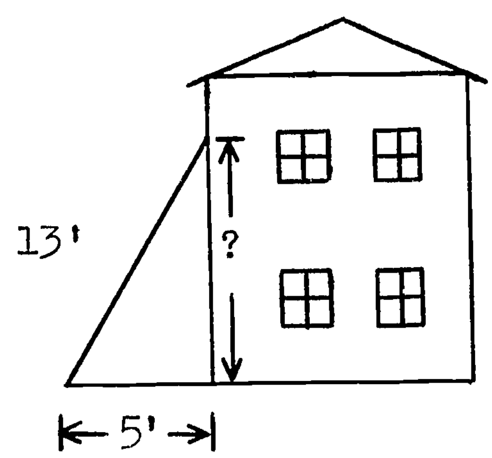
   $b^2 =$ __________

   $b =$ __________

9. $(\frac{1}{2})^2 = a^2 + (\frac{3}{10})^2$

10. $(\frac{17}{20})^2 = a^2 + (\frac{3}{4})^2$

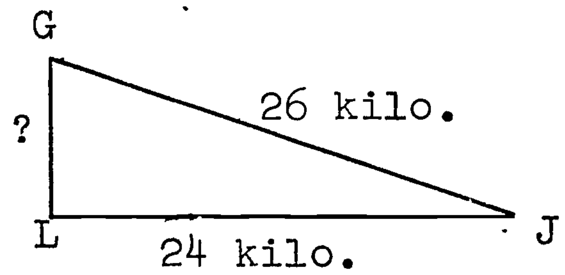
11. A 13 foot ladder is leaning against a building. The base of the ladder is 5 feet from the building. How high on the building is the end of the ladder?



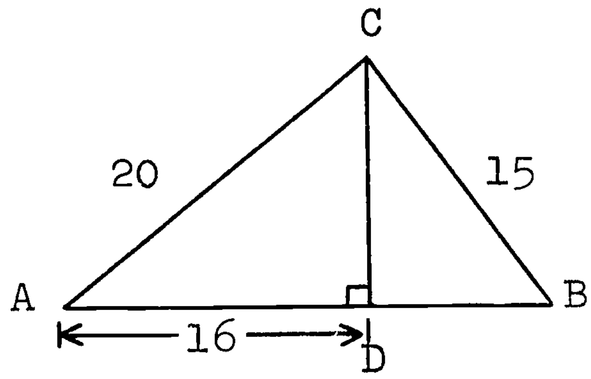
12. The length of the diagonal of a rectangle is 34 inches. The length of the rectangle has a measure of 30 inches. Find the measure of the width.

13. The length of the hypotenuse of a right triangle is 51 inches and the length of the second side is 45 inches. What is the length of the remaining side of this triangle?

14. Jack's home is almost 24 kilometers east of Larry's home and about 26 kilometers east-south-east of Gerald's home. About how far north of Larry does Gerald live?




15. An altitude of a triangle is the line segment from a vertex of an angle of the triangle perpendicular to a line which contains the side of the triangle opposite the angle. Whew! In the triangle represented here, find the length of the altitude CD.




16. Find DB of triangle ABC of problem 15. (DB is the measure of the length of the line segment DB.)

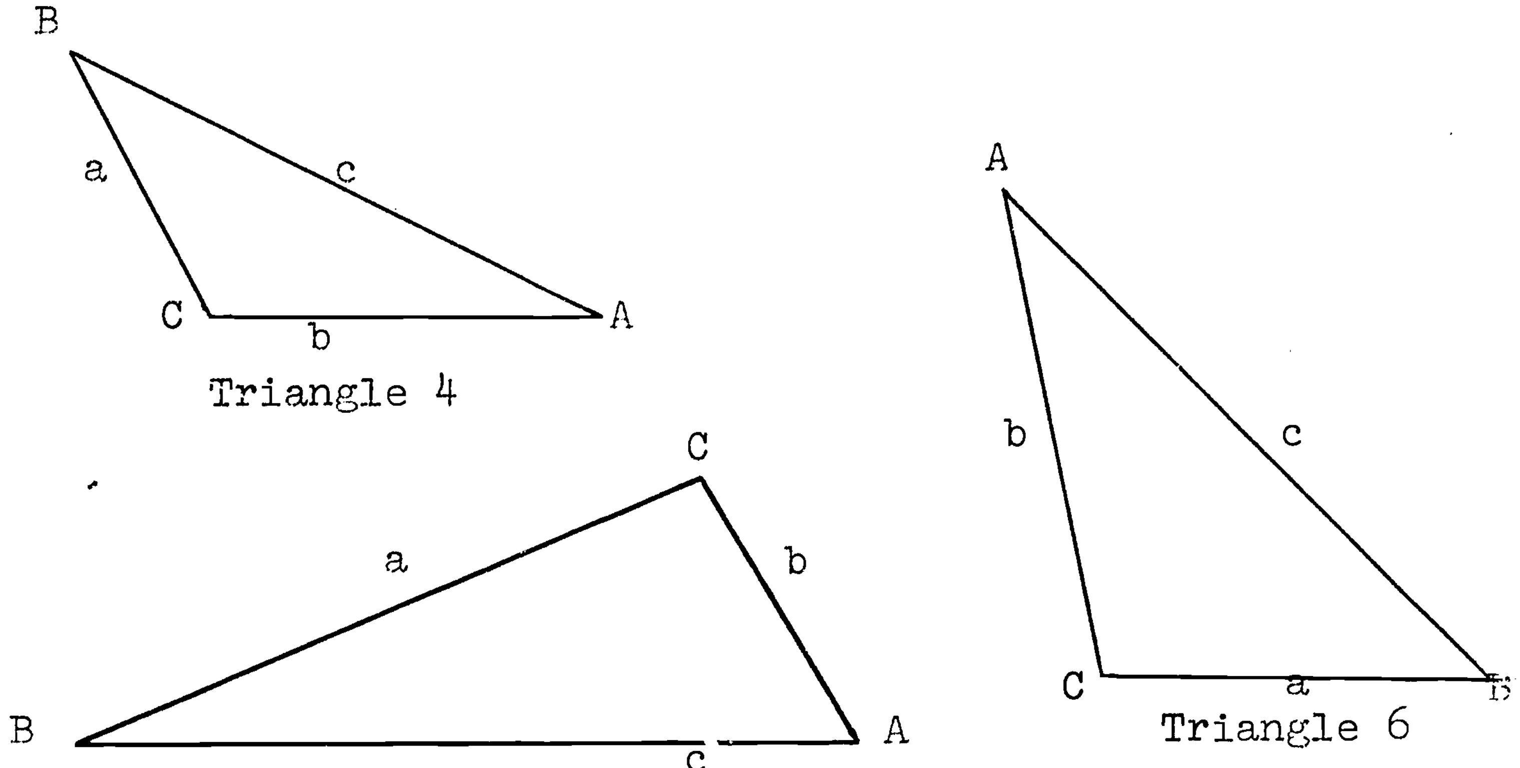
17. Is triangle ABC in problem 15 isosceles, equilateral, or scalene? ____________

18. Is ∠ACB of triangle ABC in problem 15 a right angle? __________

Do you remember that a triangle is <u>acute</u> if each of its angles has a measure less than 90, and a triangle is <u>obtuse</u> if it has exactly one angle with a measure greater than 90 but less than 180? If you do not remember these definitions, see the illustrations on page 2.

## Activities

1. Use the centimeter scale on your ruler to determine the lengths of the sides of the following triangles and then complete the table. Measure to the nearest centimeter.




Triangle 4

Triangle 5

Triangle 6

| TRIANGLE | a | b | c | $a^2$ | $b^2$ | $c^2$ | $a^2 + b^2$ |
|---|---|---|---|---|---|---|---|
| 4 | | | | | | | |
| 5 | | | | | | | |
| 6 | | | | | | | |

(Table 2)

2. Underline the correct relationship between $c^2$ and $a^2 + b^2$ of triangle 4. Use Table 2. (The symbol > means <u>greater than</u>. For instance, $6 > 5$ and $9 < 10$.)

   (a) $c^2 < a^2 + b^2$ (b) $c^2 = a^2 + b^2$

   (c) $c^2 > a^2 + b^2$

3. Is $\angle C$ of triangle 4 acute, obtuse, or right? (You may use your protractor if you need to.) __________

4. Is triangle 4 acute, obtuse, or right? __________

5. Underline the correct relationship between $c^2$ and $a^2 + b^2$ of triangle 5.

   (a) $c^2 > a^2 + b^2$

   (b) $c^2 = a^2 + b^2$

   (c) $c^2 < a^2 + b^2$

6. Is $\angle C$ of triangle 5 acute, obtuse, or right? __________

7. Is triangle 5 acute, obtuse or right? __________

8. Underline the correct relationship between $c^2$ and $a^2 + b^2$ of triangle 6.

   (a) $c^2 > a^2 + b^2$

   (b) $c^2 = a^2 + b^2$

   (c) $c^2 < a^2 + b^2$

9. Is $\angle C$ of triangle 6 acute, right, or obtuse? __________

10. Is triangle 6 acute, right, or obtuse? __________

11. Complete the following table and then use the information in it and the picture of the triangles on page 14 to answer questions 12 - 16.

| Triangle | a | b | c | $a^2$ | $b^2$ | $c^2$ | $b^2 + c^2$ | $a^2 + c^2$ |
|---|---|---|---|---|---|---|---|---|
| 4 | 3 | 4 | 6 | | | | | |
| 5 | 7 | 3 | 8 | | | | | |

( Table 3 )

12. Underline the correct relationship between $a^2$ and $b^2 + c^2$ of triangle 4.

    (a) $a^2 > b^2 + c^2$

    (b) $a^2 = b^2 + c^2$

    (c) $a^2 < b^2 + c^2$

13. Is ∠A of triangle 4 acute, obtuse, or right? __________

    Underline the correct relationship between $b^2$ and $a^2 + c^2$ of triangle 5.

    (a) $b^2 < a^2 + c^2$

    (b) $b^2 = a^2 + c^2$

    (c) $b^2 > a^2 + c^2$

14. Is ∠B of triangle 4 acute, right, or obtuse? __________

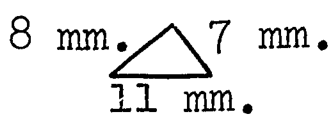
15. What is the relationship between $b^2$ and $a^2 + c^2$ of triangle 5?

    __________

16. Is ∠B of triangle 5 acute, right, or obtuse? __________

17. In the following triangle the three sides have these lengths:

    a = 7 mm.
    b = 8 mm.
    c = 11 mm.




Complete the table.

| a | b | c | $a^2$ | $b^2$ | $c^2$ | $a^2 + b^2$ | $a^2 + c^2$ | $b^2 + c^2$ |
|---|---|---|---|---|---|---|---|---|
| 7 | 8 | 11 | | | | | | |

18. Make a true statement of each of the following by placing <, = , or > in the blank. Use the results of problem 17.

    (a) $c^2$ __________ $a^2 + b^2$

    (b) $b^2$ __________ $a^2 + c^2$

    (c) $a^2$ __________ $b^2 + c^2$

19. Write the word acute, right, or obtuse in each of the blanks. Use the results of problem 18.

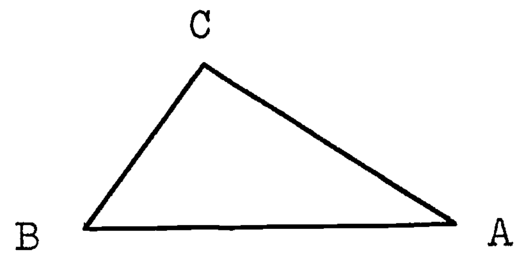
(a) Angle C is ___________

(b) Angle B is ___________

(c) Angle A is ___________

Indicate in each of the following triangles whether each angle is acute, right, or obtuse.

20.

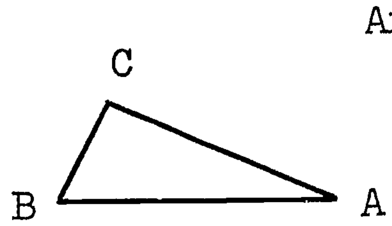



a = 20 mm. Angle A is ___________

b = 30 mm. Angle B is ___________

c = 38 mm. Angle C is ___________

21.




a = 10 mm. Angle A is ___________

b = 24 mm. Angle B is ___________

c = 26 mm. Angle C is ___________

22. a = 16 inches Angle A is ___________

b = 18 inches Angle B is ___________

c = 24 inches Angle C is ___________

23. a = 8 centimeters Angle A is ___________

b = 9 centimeters Angle B is ___________

c = 13 centimeters Angle C is ___________

24. a = $\frac{3}{10}$ inch Angle A is ___________

b = $\frac{2}{5}$ inch Angle B is ___________

c = $\frac{1}{2}$ inch Angle C is ___________

In each of the problems 25 - 28 the lengths of the three sides of a triangle are given. Indicate whether each triangle is acute, right, or obtuse.

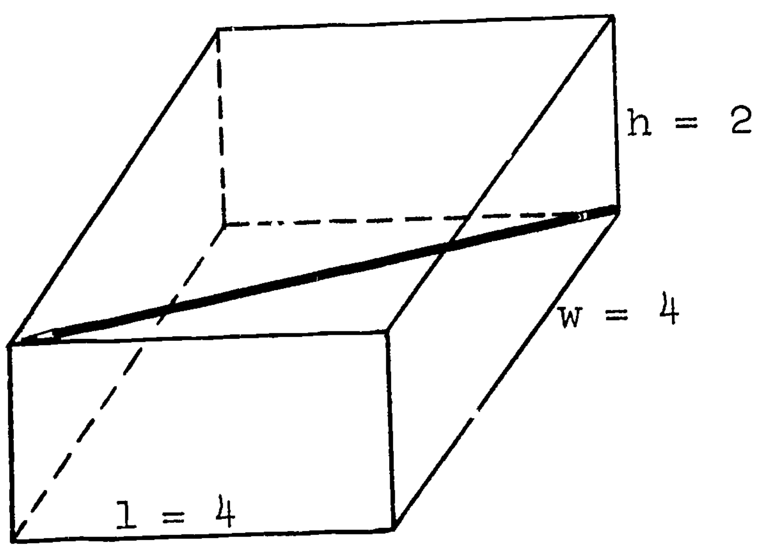
25. 4, 8, and 10 yards ___________

26. 5, 5, and 7 meters___________

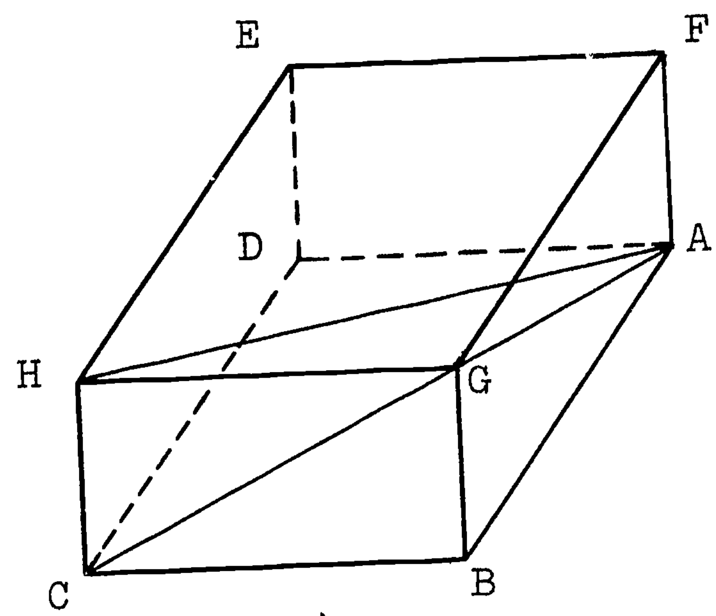
27. 9, 12, and 15 inches ___________

28. 5, 12, and 13 inches ___________

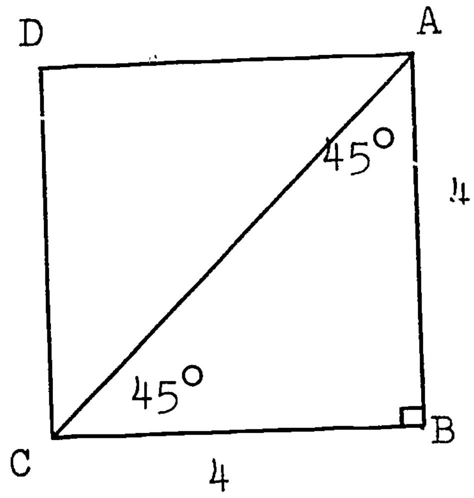
Will a pencil $5\frac{1}{2}$ inches long fit into a box which measures 4 inches by 4 inches by 2 inches?




(Figure 1)

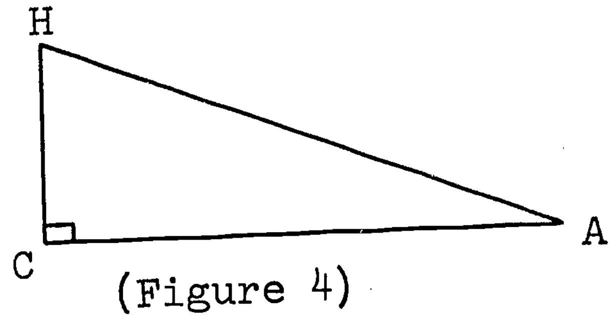



(Figure 2)




Bottom of Box

(Figure 3)




(Figure 4)

To solve this problem find the length of $\overline{AH}$. (See figures 2 and 4.) First, AC must be determined. From figure 3 it is seen that $\overline{AC}$ is the diagonal of a rectangle and that:

$$(AC)^2 = 4^2 + 4^2$$

$$(AC)^2 = 16 + 16$$

$$(AC)^2 = 32$$

From figure 4 it may now be seen that:

$$(AH)^2 = 2^2 + 32$$

$$(AH)^2 = 4 + 32$$

$$(AH)^2 = 36$$

$$AH = 6$$

Do you see that since $6 > 5\frac{1}{2}$, the pencil will fit into the box?

The length AH is known as the length of the diagonal, $\overline{AH}$, of the rectangular solid. If "d" is the measure of the length of the diagonal and "l," "w," and "h" are the measures of the length, width, and height of the rectangular solid:

$$d = \sqrt{l^2 + w^2 + h^2}$$

## Activities

The height, length, and width of rectangular solids are given in each of the following problems. Find

(a) the square of the length of the diagonal of the bottom of the rectangular solid, and

(b) the length of the diagonal of the rectangular solid.

1. 3 feet, 6 feet, and 6 feet

2. 9 centimeters, 18 centimeters, and 18 centimeters

3. 7 inches, 14 inches, and 14 inches

4. 1 inch, 2 inches, and 2 inches

5. 10 millimeters, 12 millimeters, and 15 millimeters

Give your answer to problem 5 to the nearest tenth of a millimeter.

<u>Example</u>: Will a pencil 7 inches long fit into a box with these dimensions?

length = 3 inches

width = 4 inches

height = 5 inches

<u>Solution</u>: The square of the length of the diagonal of the rectangular solid is $3^2 + 4^2 + 5^2$ or 50. The square of the length of the pencil is $7^2$ or 49. Since $3^2 + 4^2 + 5^2 > 7^2$ $(50 > 49)$, the pencil will fit into the box.

## Activities

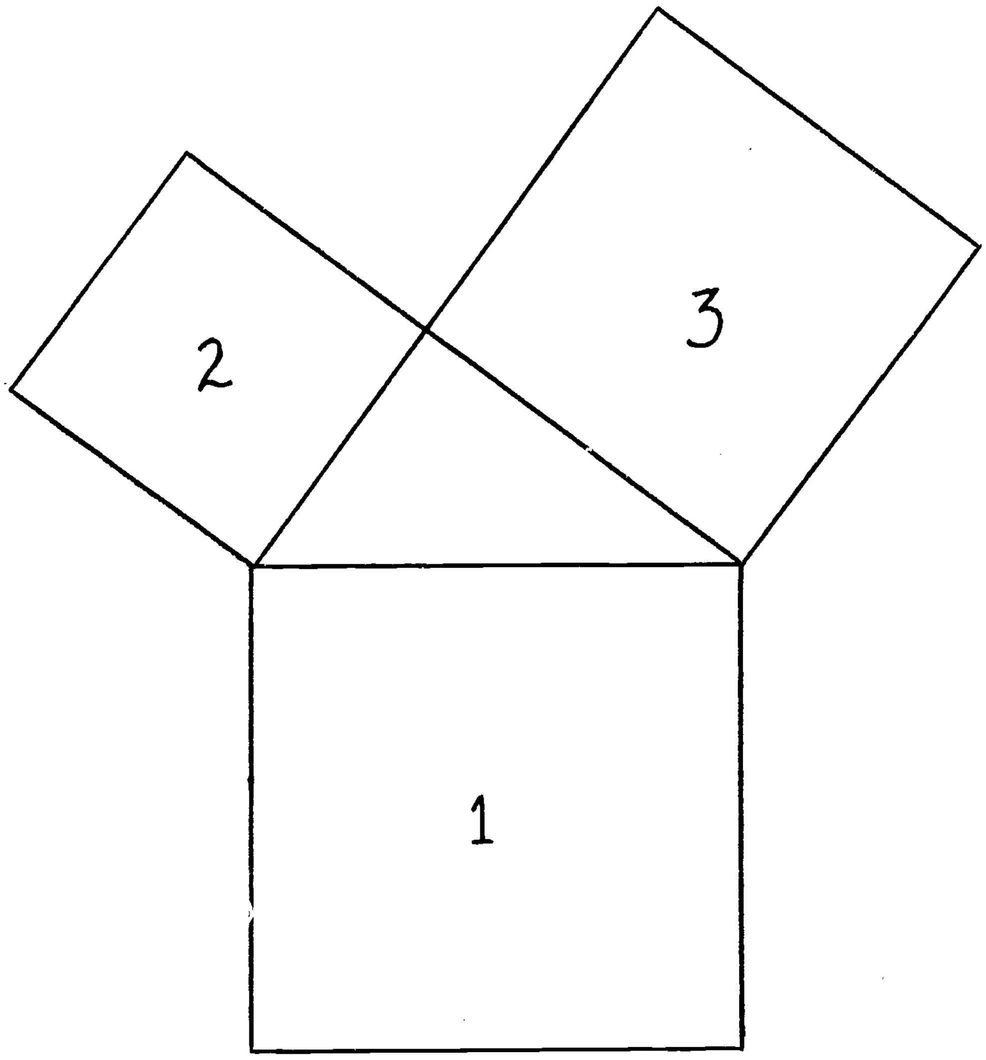
The height, length, and width of rectangular solids are given in each of the following problems. Indicate by writing YES or NO whether the object with the given length will fit into the rectangular solid.

| Rectangular Solid | | | Length of Object | YES or NO |
|---|---|---|---|---|
| height | length | width | | |
| 1. 3 in. | 4 in. | 2 in. | 6 in. | |
| 2. 9 in. | 10 in. | 13 in. | 18 in. | |
| 3. 4 in. | 4 in. | 3 in. | 6 in. | |
| 4. 15 cm. | 18 cm. | 20 cm. | 31 cm. | |
| 5. 8 in. | 12 in. | 10 in. | 18 in. | |
| 6. 2 in. | 2 in. | 2 in. | 3 in. | |
| 7. 70 mm. | 75 mm. | 75 mm. | 125 mm. | |
| 8. 97 mm. | 145 mm. | 143 mm. | 220 mm. | |
| 9. 2 in. | 5.5 in. | 3.75 in. | 7 in. | |

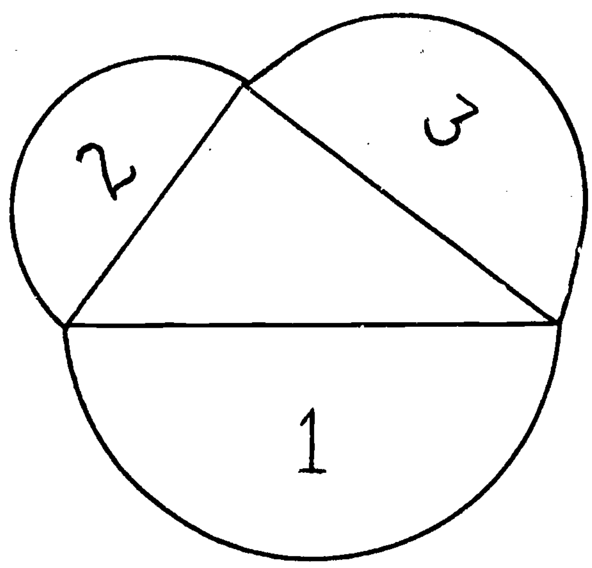
Activities

Using the centimeter scale on your ruler, determine if the measure of the area within figure 1 equals the sum of the measures of the areas within figures 2 and 3.

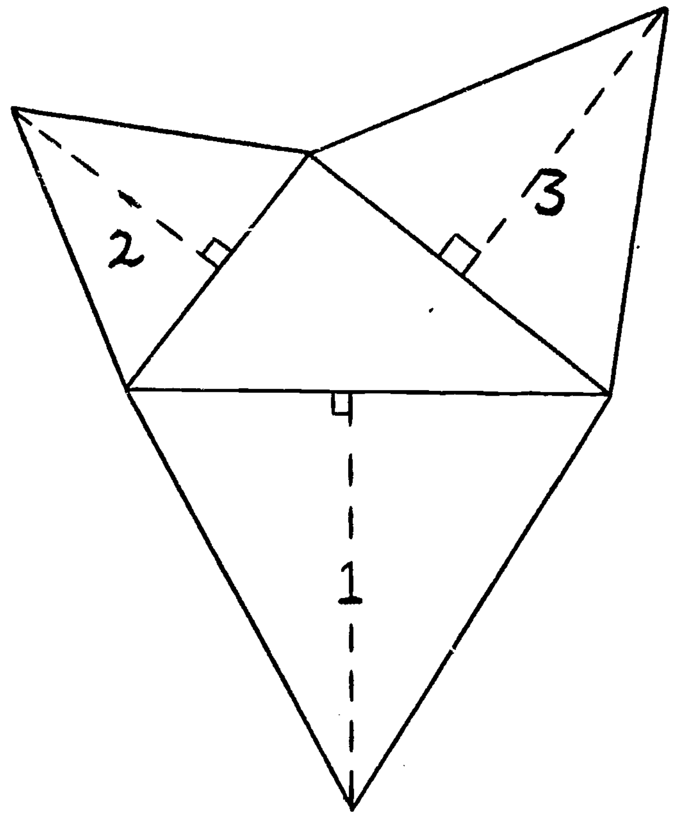
1.



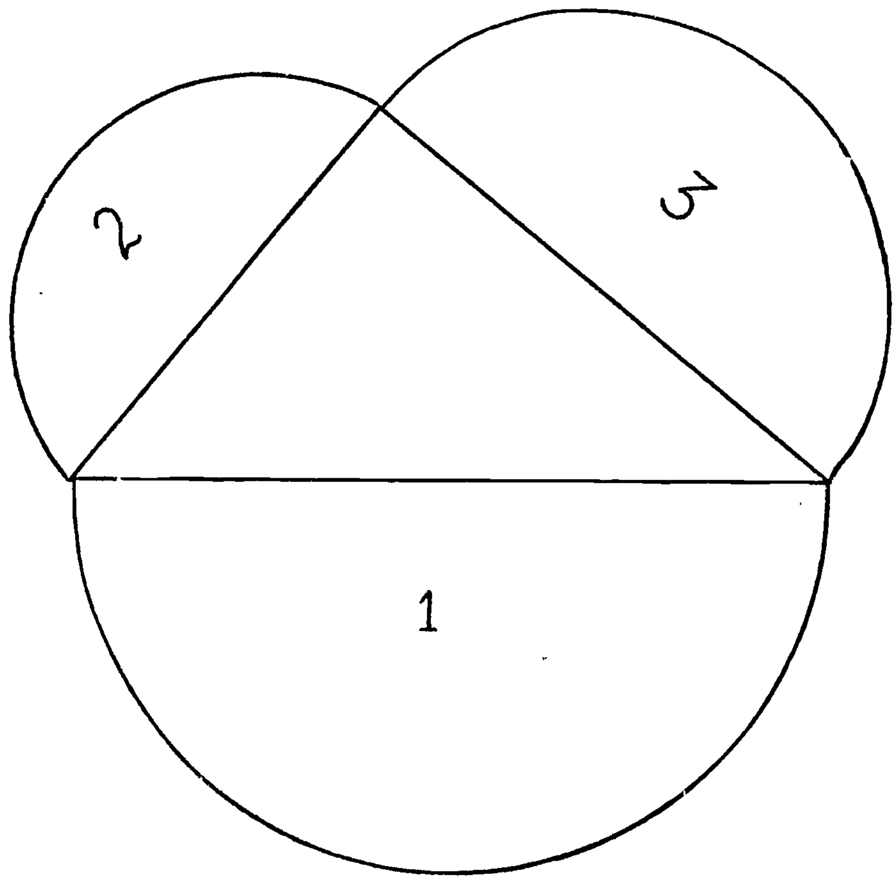

2.



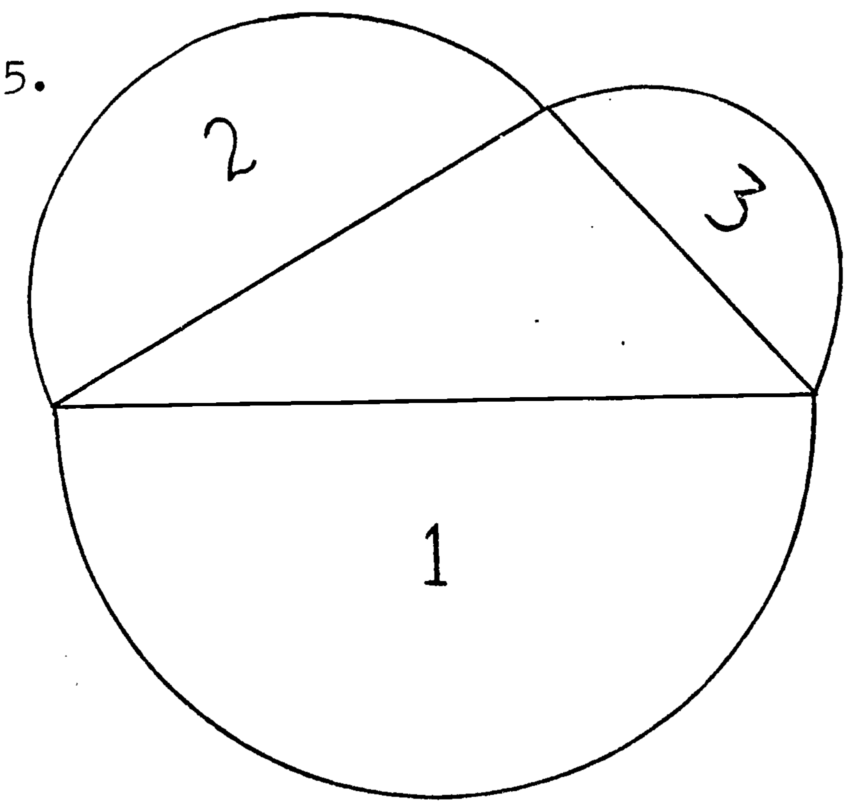
3.



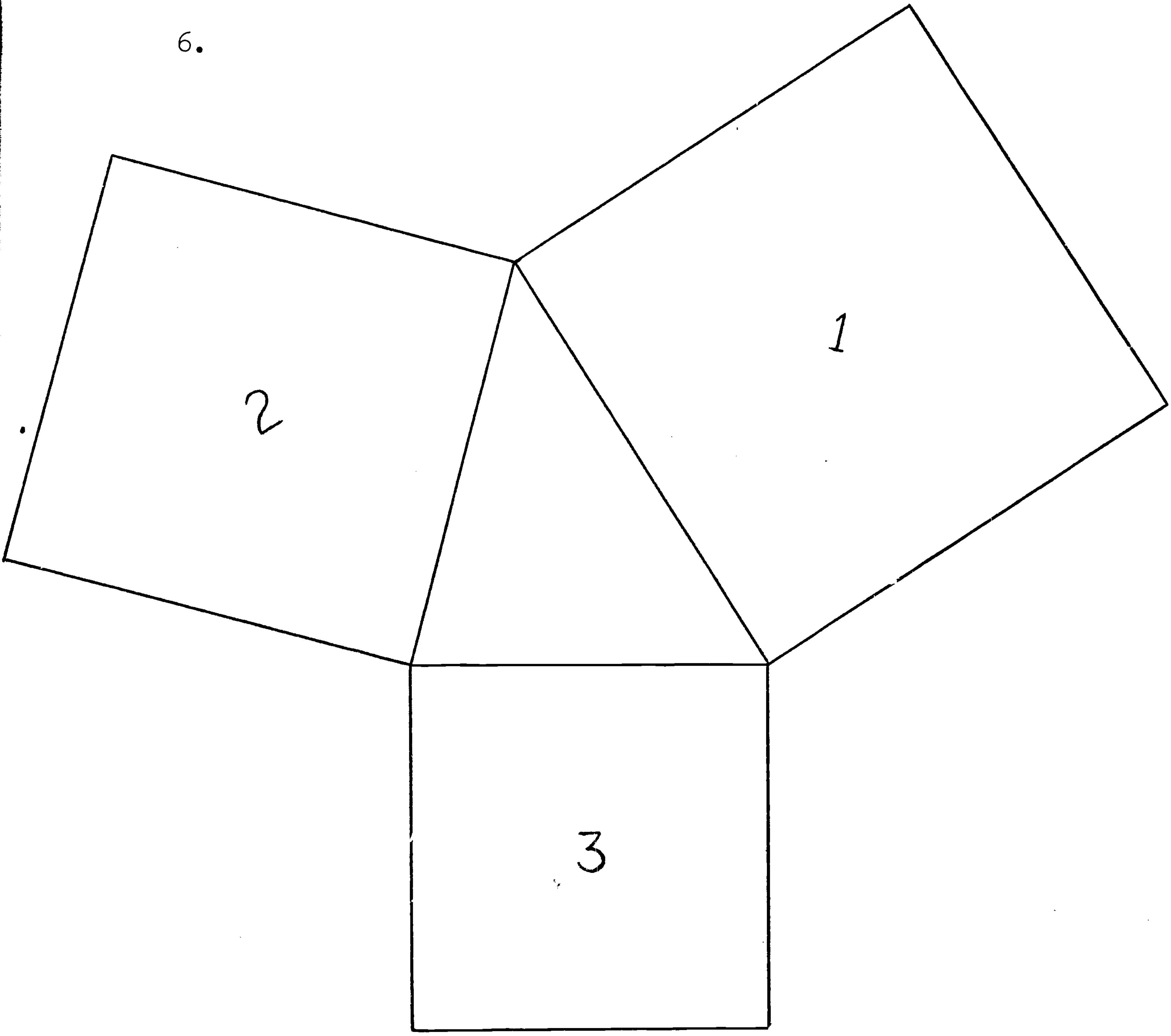

4.




5.



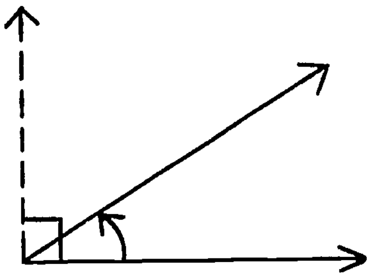
6.



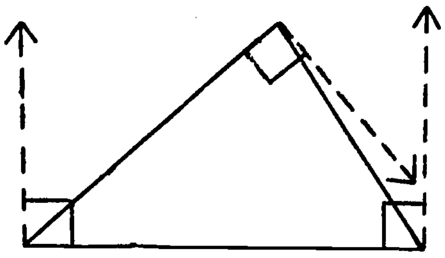
# ILLUSTRATION OF TERMS

acute, an angle whose measure is less than 90.

acute angle



acute triangle, a triangle each of whose angles is acute.



This triangle is acute since each of its angles has a measure of less than 90.

base number, number which is raised to a power

$5^2$ 5 is the base

$7^2$ 7 is the base

$1^2$ 1 is the base

compute, to determine the solution to a problem using arithmetic, algebra, etc., to determine an answer using logical operations.

diagonal, a line segment joining vertices of two nonadjacent angles of a solid or a plane figure.

distance between two points, the length of the line segment between the two points.

exponent, a number which tells how many times another number (the base) is to be used as a factor.

$5^2$ 2 is the exponent. 5 is used as a factor twice.

$5^2 = 5 \times 5 = 25$

$4^3$ 3 is the exponent. 4 is used as a factor 3 times.

$4^3 = 4 \times 4 \times 4 = 64$

<u>factors</u>, numbers listed in an indicated product.

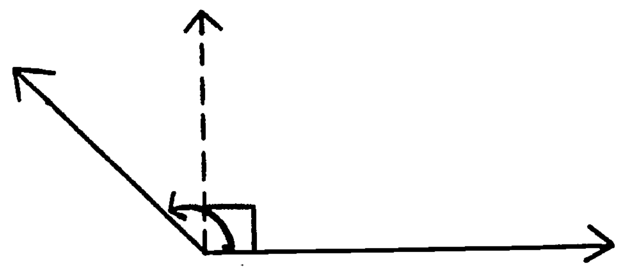
| | | |
|---|---|---|
| 16 | = 4 X 2 X 2 | 4, 2 are factors |
| 18 | = 6 X 3 | 6, 3 are factors |

<u>factor pairs</u>, when a number is factored into two other numbers, these other two numbers are called factor pairs.

| | | |
|---|---|---|
| 12 | = 4 X 3 | 4 and 3 are factor pairs |
| 12 | = 6 X 2 | 6 and 2 are factor pairs |
| 18 | = 9 X 2 | 9 and 2 are factor pairs |
| 18 | = 18 X 1 | 18 and 1 are factor pairs |

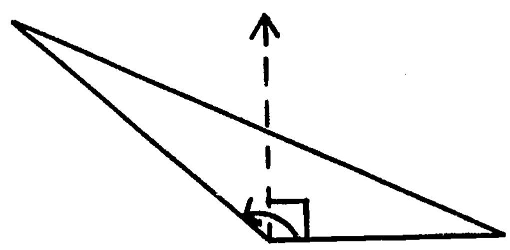
<u>hypotenuse</u>, the side of a right triangle opposite the right angle; the side of a right triangle the square of whose length equals the sum of the squares of the lengths of the other two sides; the longest side.

<u>obtuse</u>, an angle whose measure is greater than 90.



<u>obtuse</u> <u>triangle</u>, a triangle <u>one</u> of whose angles is obtuse.

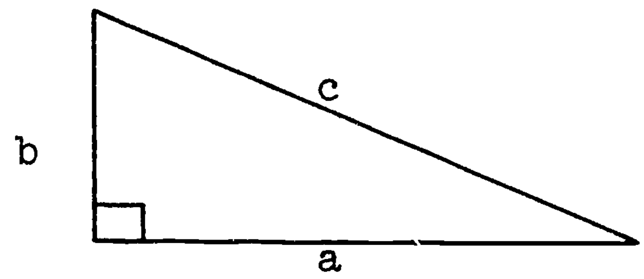
an obtuse triangle



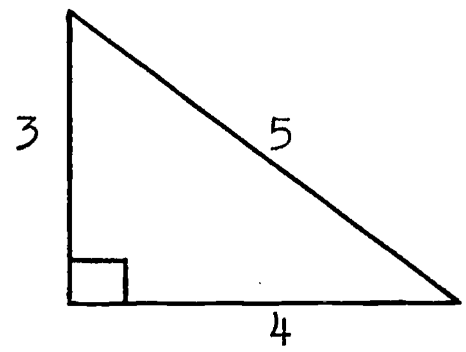
<u>product</u>, when numbers are multiplied together, the result is called the product.

| | | |
|---|---|---|
| 3 X 5 X 7 | = 105 | 105 is the product |
| 8 X 2 X 3 | = 48 | 48 is the product |
| 4 X 5 | = 20 | 20 is the product |

Pythagorean theorem, in a right triangle the square of the length of the hypotenuse equals the sum of the squares of the lengths of the other two sides.




$$c^2 = a^2 + b^2$$




$$5^2 = 3^2 + 4^2$$
$$25 = 9 + 16$$
$$25 = 25$$

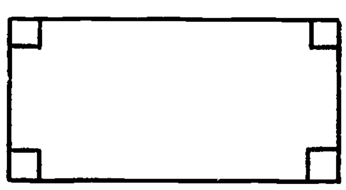
Pythagorus, a Greek mathematician and philosopher of the Sixth Century, B.C., who discovered the theorem which bears his name.

$\sqrt{\phantom{x}}$ (radical), a symbol which indicates the square root is to be found.

$$\sqrt{16} = 4$$
$$\sqrt{25} = 5$$
$$\sqrt{36} = 6$$

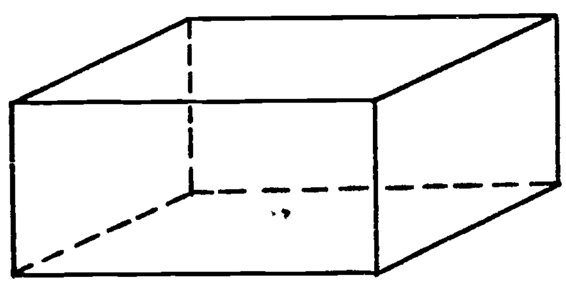
rectangle, a four-sided plane geometric figure with four right angles.

a rectangle



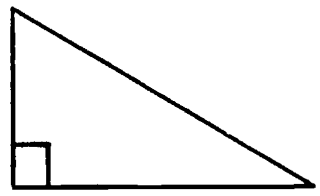
rectangular solid, a six-sided solid whose faces are all rectangles.

a rectangular solid



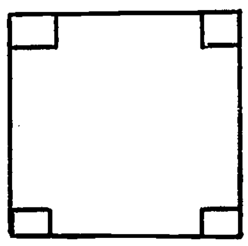
right triangle, a triangle with exactly one right angle.

a right triangle



solid (geometric), a three-dimensional geometric figure; a geometric figure having length, width, and depth. Example: cube.

square, a plane geometric figure having four sides of equal lengths and each angle has a measure of 90.



square, the number obtained by multiplying two equal numbers.

2 X 2 = 4 4 is the square of 2.

11 X 11 = 121 121 is the square of 11.

square root, one of the two equal factors of a number.

Four is the square root of 16 because 4 X 4 = 16.
Eight is the square root of 64 because 8 X 8 = 64.
Six is the square root of 36 because 6 X 6 = 36.

squaring, the multiplication of a number by itself.